Lesson 23: Problem Solving Using Rates, Unit Rates, and Conversions

Student Outcomes

Students solve constant rate work problems by calculating and comparing unit rates.

Materials

 Calculators

Classwork (30 minutes)

 If work is being done at a constant rate by one person at a different constant rate by another person, both rates can be converted to their unit rates then compared directly.
 Work can be jobs done in a certain time period, or even running or swimming rates, etc.

In the last lesson, we learned about constant speed problems. Today we will be learning about constant rate work problems. Think for a moment about what a “constant rate work” problem might be.

Allow time for speculation and sharing of possible interpretations of what the lesson title might mean. Student responses should be summarized by:

- Constant rate work problems let us compare two unit rates to see which situation is faster or slower.

Back in lesson 18 we found a rate by dividing two quantities. Recall how to do this.

1. To find a unit rate, divide the numerator by the denominator.

Example 1: Fresh-Cut Grass
Suppose that on a Saturday morning you can cut 3 lawns in 5 hours, and your friend can cut 5 lawns in 8 hours. Your friend claims he is working faster than you. Who is cutting lawns at a faster rate? How do you find out?

1. Divide the numerator by the denominator to find the unit rate.

What is 3 divided by 5?

1. 0.6

How should you label the problem?

1. The same way it is presented. Here “lawns” remains in the numerator, and “hours” remains in the denominator.

How should the fraction look when it is written completely?

\[
\frac{3 \text{ lawns}}{5 \text{ hours}} = \frac{0.6 \text{ lawns}}{1 \text{ hour}}
\]

How should it be read?

1. If I can cut 3 lawns in 5 hours, that equals \(\frac{3}{5}\) lawns in an hour. If a calculator is used, that will be a unit rate of six-tenths of a lawn in an hour.

What is the unit rate of your friend’s lawn cutting?

1. My friend is cutting \(\frac{5}{8}\) lawns in an hour.

\[
\frac{5 \text{ lawns}}{8 \text{ hours}} = \frac{0.625 \text{ lawns}}{1 \text{ hour}}
\]

How is this interpreted?

1. If my friend cuts 5 lawns in 8 hours, the unit rate is 0.625 lawns per hour.

Compare the two fractions \(\frac{3}{5}\) and \(\frac{5}{8}\).

1. \(\frac{24}{40} < \frac{25}{40}\) My friend is a little faster, but only \(\frac{1}{40}\) of a lawn per hour, so it is very close.

Benchmark fractions have corresponding decimals 0.6 and 0.625

Example 1: Fresh-Cut Grass

Suppose that on a Saturday morning you can cut \(\frac{3}{5}\) lawns in \(\frac{5}{8}\) hours, and your friend can cut 5 lawns in 8 hours. Who is cutting lawns at a faster rate?

\[
\frac{3}{5} \text{ hours} \quad \frac{5}{8} \text{ hours}
\]
My friend is a little faster, but only \( \frac{25}{40} \) of a lawn per hour, so it is very close.

Benchmark fractions have corresponding decimals 0.6 and 0.625.

### Example 2: Restaurant Advertising

Next, suppose you own a restaurant. You want to do some advertising, so you hire 2 middle school students to deliver take-out menus around town. One of them, Darla, delivers 350 menus in 2 hours, and another employee, Drew, delivers 510 menus in 3 hours. You promise a $10 bonus to the fastest worker since time is money in the restaurant business. Who gets the bonus?

- How should the fractions look when they are written completely?

\[
\frac{350 \text{ menus}}{2 \text{ hours}} = \frac{175 \text{ menus}}{1 \text{ hour}} \quad \frac{510 \text{ menus}}{3 \text{ hours}} = \frac{170 \text{ menus}}{1 \text{ hour}}
\]

Who works faster at the task and gets the bonus cash?

2. Darla.

Will the unit labels in the numerator and denominator always match in the work rates we are comparing?

3. Yes.

### Example 2: Restaurant Advertising

\[
\frac{350 \text{ menus}}{2 \text{ hours}} = \frac{175 \text{ menus}}{1 \text{ hour}} \quad \frac{510 \text{ menus}}{3 \text{ hours}} = \frac{170 \text{ menus}}{1 \text{ hour}}
\]

Set up a problem for the student that does not keep the units in the same arrangement:

\[
\frac{350 \text{ menus}}{2 \text{ hours}} = \frac{175 \text{ menus}}{1 \text{ hour}} \quad \frac{3 \text{ hours}}{510 \text{ menus}} = \frac{1 \text{ hour}}{170 \text{ menus}}
\]

What happens if they do not match, and one is inverted?

4. It will be difficult to compare the rates. We would have to say 175 menus would be delivered per hour by Darla, and it would take an hour for Drew to deliver 170 menus. Mixing up the units makes the explanations awkward.

Will time always be in the denominator?
5. Yes. Do you always divide the numerator by the denominator to find the unit rate?
6. Yes.

**Example 3: Survival of the Fittest**

Which runs faster: a cheetah that can run 60 feet in 4 seconds or gazelle that can run 100 feet in 8 seconds?

\[
\frac{60 \text{ feet}}{4 \text{ seconds}} = \frac{15 \text{ feet}}{1 \text{ second}} \quad \quad \frac{100 \text{ feet}}{8 \text{ seconds}} = \frac{12.5 \text{ feet}}{1 \text{ second}}
\]

*The cheetah runs faster.*

**Example 4: Flying Fingers**

What if the units of time are not the same in the two rates? The secretary in the main office can type 225 words in 3 minutes, while the computer teacher can type 105 words in 90 seconds. Who types at a faster rate?

Ask half of the class to solve this problem using words per minute and the other half using words per second. Ask for a volunteer from each group to display and explain their solution.

\[
\frac{225 \text{ words}}{3 \text{ minutes}} = \frac{75 \text{ words}}{1 \text{ minute}} \quad \quad \frac{105 \text{ words}}{1.5 \text{ minutes}} = \frac{70 \text{ words}}{1 \text{ minute}}
\]

*The secretary types faster.*

\[
\frac{225 \text{ words}}{180 \text{ seconds}} = \frac{1.25 \text{ words}}{1 \text{ second}} \quad \quad \frac{105 \text{ words}}{90 \text{ second}} = \frac{1.166667 \text{ words}}{1 \text{ second}}
\]

*The secretary types faster.*

Do we have to change one time unit?
7. Yes.

What happened if we do not convert one time unit so that they match?
8. We cannot compare the rates. It is not easy to tell which is faster: 70 words per minute or 1.25 words per second.

Does it matter which one you change?

9. No. Either change 90 seconds to 1.5 minutes or change 3 minutes to 180 seconds.

Can you choose the one that makes the problem easier for you?

10. Yes.

Is there an advantage in choosing one method over the other?

11. Changing seconds to minutes avoids repeating decimals.

Looking back on our work so far what is puzzling you?

Describe how this type of problem is similar to unit pricing problems.

12. Unit pricing problems use division, and so do work rate problems.

Describe how work problems are different than unit price problems?

13. Unit price problems always have cost in the numerator; work rate problems always have time in the denominator.

Closing (5 minutes)

Constant rate problems always count or measure something happening per unit of time. The time is always in the denominator.

Sometimes the units of time in the denominators of two rates are not the same. One must be converted to the other before calculating the unit rate of each.

Dividing the numerator by the denominator calculates the unit rate; this number stays in the numerator. The number in the denominator of the equivalent fraction is always 1.

Lesson Summary

- Constant rate problems always count or measure something happening per unit of time. The time is always in the denominator.
- Sometimes the units of time in the denominators of two rates are not the same. One must be converted to the other before calculating the unit rate of each.
- Dividing the numerator by the denominator calculates the unit rate; this number stays in the numerator. The number in the denominator of the equivalent fraction is always 1.

Exit Ticket (5 minutes)
Lesson 23: Problem Solving Using Rates, Unit Rates, and Conversions

Exit Ticket

A 6th grade math teacher can grade 25 homework assignments in 20 minutes.

Is this working at a faster rate or slower rate than grading 36 homework assignments in 30 minutes?
The following solutions indicate an understanding of the objectives of this lesson:

A 6th grade math teacher can grade 25 homework assignments in 20 minutes.

Is this working at a faster rate or slower rate than grading 36 homework assignments in 30 minutes?

\[
\frac{25 \text{ assignments}}{20 \text{ minutes}} = \frac{1.25 \text{ assignments}}{1 \text{ minute}}
\]

\[
\frac{36 \text{ assignments}}{30 \text{ minutes}} = \frac{1.2 \text{ assignments}}{1 \text{ minute}}
\]

It is faster to grade 25 assignments in 20 minutes.

Problem Set Sample Solutions

Who walks at a faster rate: someone who walks 60 feet in 10 seconds or someone who walks 42 feet in 6 seconds?

\[
\frac{60 \text{ feet}}{10 \text{ sec}} = 6 \frac{\text{ft}}{\text{sec}}
\]

\[
\frac{42 \text{ ft}}{6 \text{ sec}} = 7 \frac{\text{ft}}{\text{sec}} \text{ Faster}
\]

Who walks at a faster rate: someone who walks 60 feet in 10 seconds or someone who takes 5 seconds to walk 25 feet? Review the lesson summary before answering!

\[
\frac{60 \text{ ft}}{10 \text{ sec.}} = 6 \frac{\text{ft}}{1 \text{ sec}} \text{ Faster}
\]

\[
\frac{25 \text{ ft}}{5 \text{ sec}} = 5 \frac{\text{ft}}{\text{sec}}
\]

Which parachute has a slower decent: a red parachute that falls 10 feet in 4 seconds or a blue parachute that falls 12 feet in 6 seconds?
\[ \frac{10 \text{ ft}}{4 \text{ sec}} = 2.5 \text{ ft/sec} \]

\[ \frac{12 \text{ ft}}{6 \text{ sec}} = 2 \text{ ft/sec} \text{ Slower} \]

During the winter of 2012-2013, Buffalo, New York received 22 inches of snow in 12 hours. Oswego, New York received 31 inches of snow over a 15 hour period. Which city had a heavier snowfall rate? Round your answers to the nearest hundredth.

\[ 22 \in \frac{\text{i}}{12 \text{ hrs}} = 1.83 \frac{\text{i}}{\text{hr}} \text{ Heavier} \]

\[ 31 \in \frac{\text{i}}{15 \text{ hrs}} = 2.07 \frac{\text{i}}{\text{hr}} \]

A striped marlin can swim at a rate of 70 miles per hour. Is this a faster or slower rate than a sailfish, which takes 30 minutes to swim 40 miles?

\[ \text{Marlin:} \quad 70 \text{ mph} \quad \text{slower} \]

\[ \text{Sailfish:} \]

\[ \frac{40 \text{ miles}}{30 \text{ minutes}} \times \frac{60 \text{ minutes}}{1 \text{ hr}} = \frac{2400 \text{ miles}}{30 \text{ hours}} = 80 \text{ mph} \]

One math student, John, can solve these 6 math problems in 20 minutes while another student, Juaquine, can solve them at a rate of 1 problem per 4 minutes. Who works faster?

\[ \frac{6 \text{ problems}}{20 \text{ minutes}} = 0.3 \text{ problems/minute} \text{ Faster} \]

\[ \frac{1 \text{ problem}}{4 \text{ minutes}} = 0.25 \text{ problems/minute} \]