PRACTICE TASK: Investigating Quadrilaterals

In this task, students will investigate the attributes of quadrilaterals.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.G.3 Understanding that attributes belonging to a category of two-dimensional figures also belong to all subcategories.

MCC5.G.4 Classify two-dimensional figures in a hierarchy based on properties.

STANDARDS FOR MATHEMATICAL PRACTICE

SMP 1. Make sense of problems and persevere in solving them.
SMP 2. Reason abstractly and quantitatively.
SMP 3. Construct viable arguments and critique the reasoning of others.
SMP 5. Use appropriate tools strategically.
SMP 6. Attend to precision.
SMP 7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

Students should have the following background knowledge.

- Be able to use a straight edge or ruler to draw a straight line.
- Know how to use a protractor, a ruler, and how to identify right angles (90 degrees), obtuse angles, and acute angles (using a protractor or the corner of an index card).
- Understand that opposite sides can not touch each other; they are on opposite sides of the quadrilateral.
- Know parallel means that lines will never intersect or cross over each other no matter how long they are extended. (Students may prove that lines are parallel by laying down 2 straight objects, such as rulers, on the parallel sides of the quadrilateral, extending those sides. This will show how the line segments do not intersect even if they are extended.)
- Understand that perpendicular means lines or segments intersect or cross forming a right angle. (Some students may use a protractor, while others may use the corner of an index card or the corner of a sheet of paper to show an angle is a right angle.)
- Know that a property is an attribute of a shape that is always going to be true. It describes the shape.
- Be able to use a ruler to measure sides to verify they are the same length.
- Be able to use a mirror to check lines of symmetry
- Be able to use tracing paper to check for angle congruence

Some properties of quadrilaterals that should be discussed are included below. As students draw conclusions about the relationships between different figures, be sure they are able to explain their thinking and defend their conclusions.
• A shape is a quadrilateral when it has exactly 4 sides and is a polygon. (To be a polygon the figure must be a closed plane figure with three or more straight sides.)
• A square is always a rectangle because a square will always have 4 right angles like a rectangle.
• A rectangle does not have to have 4 equal sides like a square. It can have 4 right angles without 4 equal sides. Therefore, rectangle is not always a square.
• A square is always a rhombus because it has 4 equal sides like a rhombus and it is also a rectangle because it has 4 right angles like a rectangle.
• A rhombus does not have to have right angles like a square. It can have 4 equal sides without having 4 right angles. Therefore a rhombus is not always a square.
• A parallelogram can be a rectangle if it has 4 right angles.

COMMON MISCONCEPTIONS
Students think that when describing geometric shapes and placing them in subcategories, the last category is the only classification that can be used.

ESSENTIAL QUESTIONS
• How can I compare and contrast the different quadrilaterals?
• What is the rationale for grouping the quadrilaterals together?

MATERIALS
• Chart Paper
• Re-sealable plastic bag per student
• Attributes of Quadrilateral sheet per student
• Set of Quadrilateral Shapes per pair of students
• Overhead transparency

GROUPING
whole/pairs/ individual task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION
Students typically enter fourth grade able to identify several shapes. Identifying shapes, however, does not provide sufficient foundations for the higher levels of reasoning required in later grades. Instruction in identifying specific classes of quadrilaterals and in understanding the hierarchy of quadrilaterals does help students move to a higher level of reasoning about two-dimensional figures. (Adding It Up: Helping Children Learn Mathematics) While this quote refers to fourth graders, it applies to fifth graders as well.

Pre-Assessment
• Have students complete a quick-write exercise in a journal explaining what they know about quadrilaterals.
• Select students to share what they know about quadrilaterals with the class and record the information on the board or chart paper.
• Ask students what questions they have about quadrilaterals and record those on chart paper.
• Use a K-W-L chart as an alternative

Scoring Guidelines for K-W-L or Quick Responses
Informally assess the students’ responses. Possible responses include:
• names of quadrilaterals (square, rectangle, rhombus, parallelogram and trapezoid) and if they know defining qualities
• quantitative descriptions such as the number of sides and vertices
• qualitative descriptions such as types of angles (acute, right, obtuse) and line relationships (parallel or perpendicular)

Day One
1. Distribute Attributes of Quadrilaterals, Attachment B and Quadrilateral Cards, Attachment C to pairs of students and have students cut out the twelve cards.
2. Use the information about quadrilaterals in the pre-assessment and observations students make using the cards to complete the first column, Quadrilaterals, on Attributes of Quadrilaterals, Attachment B. Depending on depth of prior knowledge, as revealed in pre-assessment, choose to have partners complete the column or complete the column as a class.
3. Have students sort the cards by the shapes at the top of Attributes of Quadrilaterals, Attachment B. Observe students as they sort the cards and provide assistance as needed.
4. Have students complete the chart together, using what they know about the shapes and the cards.
5. Complete a class chart on the overhead and allow students to make changes to their own charts. An example of a completed chart is provided on page two of Attributes of Quadrilaterals, Attachment B.
6. Have partners sort the shapes using different attributes. They may choose to use the attributes listed on Attributes of Quadrilaterals, Attachment B. Then, select students to share the attribute they used to sort the shapes and present the sorting to the class.
7. Have students store their shape cards in re-sealable bags for the next lesson.

Day Two
8. Have students take out the shape cards and Attributes of Quadrilaterals, Attachment B.
9. Have the students use the chart to compare the quadrilaterals. Have them create a list of attributes shapes share. For example: All sides of the square and rhombus are congruent.
10. Select students to share their comparison statements with the class.
11. Explain to students that by sorting shapes additional comparisons can be made and relationships among the shapes can be revealed.
12. Have the class sort the shapes into two categories, shapes with parallel sides and shapes without parallel sides.
13. Observe students as they sort the shapes and assist as necessary. Reinforce the concept of parallel lines.
14. Select students to present the sorting and allow other students to provide feedback.
15. Direct the students to the pile of shapes with parallel sides. Have pairs sort the shapes into two piles, shapes with one set of parallel sides and shapes with two sets of parallel sides. Observe students as they sort and provide assistance as necessary.
16. Select students to present the sorted shapes. Have students identify the names of the shapes in each pile.
17. Explain to the students that trapezoids have at least one set of parallel sides and that parallelograms have two sets of parallel sides. Ask students to identify the shapes in the parallelogram pile. Tell students that squares, rectangles and rhombi are special parallelograms.
18. Have partners sort the parallelograms by angle measure, shapes with right angles and shapes without right angles. Observe how pairs sort the shapes and provide assistance as needed.
19. Have students identify the shapes in each pile. Ask students questions about the relationships.
   • What do squares and rectangles have in common? (four right angles)
   • How are the rectangles and squares on the cards different? (lengths of sides)
   • Can a square be described as a rectangle? Why? (A square is a special rectangle with four congruent sides.)
20. Have students compare a rhombus and a square. Ask questions and allow pairs to discuss before selecting students to respond. Questions for discussion include:
   • How are the square and rhombus alike?
   • How are the rhombi and squares on the cards different?
   • Can a square be described as a rhombus? Why? (A square is a special rhombus with four right angles.)
21. Summarize the relationships visually using a graphic organizer. Quadrilateral Relationships, Attachment D is an example of an appropriate organizer. Distribute to the students and complete the organizer together. Encourage students to use the shape cards to re-sort the shapes with the attributes used during the lesson.

Instructional Tips:
• In the following days, have students share the graphic organizer and write comparisons of the quadrilaterals.
For morning work or problem of the day, present prompts for students to respond, about quadrilateral relationships. Prompts may include:

1. All squares are rectangles, but not all rectangles are squares. Why?
2. All squares are rhombi, but not all rhombi are squares. Why?
3. Squares, rectangles and rhombi are parallelograms. Why?

Post – Assessment
Use Attachment A, Quadrilateral Post-Assessment. Given a word bank of quadrilaterals, students select two figures to compare and contrast in a Venn diagram. Students then select two different quadrilaterals to compare and contrast in a table.

FORMATIVE ASSESSMENT QUESTIONS
The following questions are provided for teacher reflection since this task is already an assessment.

- How do you know what attributes are important when comparing quadrilaterals?
- How did you decide to sort your shapes? What did you think about?
- How did you choose which quadrilaterals to compare?
- Can you compare two different quadrilaterals? What will change?

DIFFERENTIATION

Extension
- Make a class dictionary on the quadrilaterals and the vocabulary terms studied.
- Provide students with shapes that include polygons other than quadrilaterals such as pentagons, hexagons and different kinds of triangles. Place shapes in an envelope. Have students sort them into 2 groups and explain why some shapes fit in one group and why others are left out of that group. Have students sort two to three times.

Intervention
- Use two-dimensional manipulatives or geo-boards to investigate the properties, make conjectures and draw conclusions on quadrilaterals.

- Have students create a game board using the two-dimensional shapes with game cards asking questions identifying the shapes, and stating questions with answers on their similarities and differences. Such questions may be: How is a square similar to a rectangle? How is a rhombus like a parallelogram? Why do some trapezoids not fit in with the parallelogram, rectangle, rhombus, and square?
Home Connection:

- Assign an interdisciplinary activity. Assign homework where the student will draw a design or creature with the shapes: trapezoid, parallelogram, rectangle, rhombus, and a square. When the student returns his creative quadrilateral creature, provide them with an overhead transparency to draw a habitat for the creature. Place the overhead transparency over top of the creature or animal and share during Science class.

- Have the students communicate with their families the similarities and differences of quadrilaterals: trapezoid, parallelogram, rectangle, the rhombus and square.

- Have students build models of quadrilaterals out of household materials such as toothpicks, cotton swabs, spaghetti, and pieces of yarn or string. Include a writing portion of the assignment in which students describe the quadrilateral.

TECHNOLOGY CONNECTION

- [http://teams.lacoe.edu/documentation/classrooms/amy/geometry/6-8/activities/quad_quest/quad_quest.html](http://teams.lacoe.edu/documentation/classrooms/amy/geometry/6-8/activities/quad_quest/quad_quest.html) Quadrilateral Quest
Directions: From the list of quadrilaterals, select two. Use the Venn diagram to compare and contrast them. List at least two similarities and two differences for each.

Name of Quadrilateral 1________________      Name of Quadrilateral 2________________

Directions: Complete this activity again using two different quadrilaterals. Use the table to compare and contrast them. List at least two similarities and two differences for each.

Name of Quadrilateral 3________________      Name of Quadrilateral 4________________

<table>
<thead>
<tr>
<th>Common Characteristics</th>
<th>Different Characteristics</th>
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<tbody>
<tr>
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### Investigating Quadrilaterals - Attachment B: Attributes of Quadrilaterals

<table>
<thead>
<tr>
<th>Attributes of Shapes</th>
<th>Quadrilateral</th>
<th>Square</th>
<th>Rectangle</th>
<th>Rhombus</th>
<th>Trapezoid</th>
<th>Parallelogram</th>
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</thead>
<tbody>
<tr>
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<td></td>
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<td></td>
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<tr>
<td>Number of angles</td>
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</tr>
<tr>
<td>Congruent sides</td>
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<tr>
<td>Congruent angles</td>
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<tr>
<td>Right angles</td>
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<tr>
<td>Parallel sides</td>
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<tr>
<td>Symmetry</td>
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<tr>
<td>Congruent angles</td>
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## Attributes of Quadrilaterals

<table>
<thead>
<tr>
<th>Attributes of Shapes</th>
<th>Quadrilateral</th>
<th>Square</th>
<th>Rectangle</th>
<th>Rhombus</th>
<th>Trapezoid</th>
<th>Parallelogram</th>
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</thead>
<tbody>
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<td>Number of sides</td>
<td>4</td>
<td>4</td>
<td>4</td>
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<tr>
<td>Number of angles</td>
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<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Congruent sides</td>
<td>Does not have to have congruent sides</td>
<td>All sides are congruent</td>
<td>Opposite sides are congruent</td>
<td>All sides are congruent</td>
<td>Does not have to have congruent sides</td>
<td>Opposite sides are congruent</td>
</tr>
<tr>
<td>Congruent angles</td>
<td>Does not have to have congruent angles</td>
<td>All angles are congruent</td>
<td>All angles are congruent</td>
<td>Opposite angles are congruent</td>
<td>Does not have to have congruent angles</td>
<td>Opposite angles are congruent</td>
</tr>
<tr>
<td>Right angles</td>
<td>Does not have to have a right angle</td>
<td>All angles are right angles</td>
<td>All angles are right angles</td>
<td>Does not have to have right angles</td>
<td>Does not have to have right angles</td>
<td>Does not have to have right angles</td>
</tr>
<tr>
<td>Parallel sides</td>
<td>Does not have to have parallel sides</td>
<td>Opposite sides are parallel</td>
<td>Opposite sides are parallel</td>
<td>Opposite sides are parallel</td>
<td>At least one set of opposite sides are parallel</td>
<td>Opposite sides are parallel</td>
</tr>
<tr>
<td>Symmetry</td>
<td>Does not have to have symmetry</td>
<td>Has four lines of symmetry</td>
<td>Has at least two lines of symmetry</td>
<td>Has at least two lines of symmetry</td>
<td>Does not have to have a line of symmetry</td>
<td>Does not have to have symmetry</td>
</tr>
<tr>
<td>Congruent angles</td>
<td>Does not have to have congruent angles</td>
<td>All angles are congruent</td>
<td>All angles are congruent</td>
<td>Opposite angles are congruent</td>
<td>Does not have to have congruent angles</td>
<td>Opposite angles are congruent</td>
</tr>
</tbody>
</table>
Attachment C: Quadrilateral Shapes

1

2

3

4

5

6
Attachment C (continued): Quadrilateral Cards
Attachment D: Quadrilateral Relationships

Note to teachers: you do not need to use the term trapezium. It is the same as trapezoid. You also do not need to introduce students to isosceles trapezoids at this point. For the purposes of this standard, use the general category of trapezoids.
### Quadrilateral - 4 sided figure

<table>
<thead>
<tr>
<th>Quadrilateral Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trapezoid</td>
<td>4 sided and at least 1 pair of parallel sides</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>All of the attributes of a trapezoid, and 2 pairs of parallel sides (which results in congruent opposite angles)</td>
</tr>
<tr>
<td>Rectangle</td>
<td>All of the attributes of the parallelogram, and 4 right angles</td>
</tr>
<tr>
<td>Square</td>
<td>All of the attributes of parallelogram, rhombus, and rectangle, and equal sides, equal angles</td>
</tr>
<tr>
<td>Kite</td>
<td>4 sided and pairs of congruent adjacent sides</td>
</tr>
<tr>
<td>Rhombus</td>
<td>All of the attributes of a parallelogram and kite</td>
</tr>
</tbody>
</table>

This hierarchy with definitions is for teacher understanding only. Please use it as a guide for your thinking when questioning students or supporting development of student understanding through the tasks.