CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Rational and Irrational Numbers 2

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Beta Version

Rational and Irrational Numbers 2

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students reason about the properties of rational and irrational numbers. In particular, this unit aims to help you identify and assist students who have difficulties in:

- Finding irrational and rational numbers to exemplify general statements.
- Reasoning with properties of rational and irrational numbers.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards* for Mathematics:

N-RN: Use properties of rational and irrational numbers.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*:

3. Construct viable arguments and critique the reasoning of others.

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students attempt the assessment task individually. You review students' work and formulate questions that will help them improve their solutions.
- During the lesson, students work collaboratively in small groups, reasoning about general statements on rational and irrational numbers. Then, in a whole class-discussion, students explain and compare different justifications they have seen and used for making their classification decisions.
- Finally, students again work alone to improve their individual solutions to the assessment task.

MATERIALS REQUIRED

- Each student will need a mini-whiteboard, an eraser, a pen, a copy of the assessment task *Rational or Irrational?,* and a copy of the review task *Rational or Irrational? (revisited)*.
- For each small group of students provide a copy of the task sheet *Always, Sometimes or Never True,* a copy of the sheet *Poster Headings*, scissors, a large sheet of paper, and a glue stick.
- Have several copies of the sheets *Hints*: *Rational and Irrational Numbers* and *Extension Task* for any students who need them, and calculators for those who wish to use them.
- There are some projectable resources with task instructions and to help support discussion.

TIME NEEDED

15 minutes before the lesson for the assessment task, a 60-minute lesson, and 10 minutes after the lesson or for homework. All timings are approximate, depending on the needs of your students.

BEFORE THE LESSON

Assessment task: *Rational or Irrational?* (15 minutes)

Have students work on this task in class or for homework a few days before the formative assessment lesson. This will give you an opportunity to assess the work and should help you identify how to help students improve their work.

Give out the task *Rational or Irrational?* Introduce the task briefly and help the class to understand the activity.

> *Spend ten minutes on you own, answering these questions.*
>
> *Show all your calculations and reasoning on the sheet.*
>
> *I have calculators if you want to use one.*

It is important that, as far as possible, students answer the questions without assistance.

Rational or Irrational?

1. a. Write three rational numbers.

   ....

   b. Explain what a rational number is, in your own words.

   ....

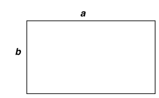
2. a. Write three irrational numbers.

   ....

   b. Explain what an irrational number is, in your own words.

   ....

3. This rectangle has sides lengths a and b.

   Decide if it is possible to find a and b to make the statements below true.

   If you think it is possible, give values for a and b.

   If you think it is impossible, explain why no values of a and b will work.

   a. The perimeter and area are both rational numbers.

   ....

   b. The perimeter is an irrational number, and the area is a rational number.

   ....

   c. The perimeter is a rational number, and the area is an irrational number.

   ....

   d. The perimeter and area are both irrational numbers.

   ....

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task that should help them to improve. Explain to students that, by the end of the next lesson, they should expect to answer questions such as these with confidence.

Assessing students' responses

Collect students' written work. Read through their scripts and make informal notes on what the work reveals about their difficulties with the math. The purpose of this is to forewarn you of issues that may arise during the lesson so that you may prepare carefully.

We strongly suggest that you do not score students' work, as research shows that this is counterproductive. It encourages students to compare grades and distracts their attention from what they are to do to improve their work.

Instead, you can help students make progress by asking questions that focus attention on aspects of their mathematics. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write your own lists of questions, based on your own students' work, using the ideas below. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the end of the lesson.

Common issues:	Suggested questions and prompts:
Student does not properly distinguish between rational and irrational numbers (Q1, Q2) For example: The student does not write examples fitting one/both categories. Or: The student does not provide definitions of one/both categories. Or: The student gives the partial, incorrect definition that a rational number can be written as a fraction.	• A rational number can always be written as a fraction of integers. What kind of decimal representations can fractions have? • An irrational number can never be written as a fraction of integers. It always has a non-repeating non-terminating decimal. Which numbers do you know that have these properties? • Which square roots are rational numbers? Which are irrational?
Student does not attempt the question (Q3) For example: The student does not identify relevant formulae to use.	• How do you calculate the area of a rectangle? Now try some numbers in your formulas. What is the perimeter of your rectangle?
Student does not provide examples (Q3)	• Figure out some values for a and b to support your explanation.
Student does not identify appropriate examples For example: The student provides some examples but draws on a limited range of numbers.	• Suppose the rectangle were square. What area might the square have? What might the side lengths be? What would the perimeter be, then? • What irrational numbers do you know? Try some out with your formulas.
Empirical reasoning (Q3) For example: The student provides examples that show the statement is not true, and concludes that there are no values of a, b that make the statement true.	• What would you have to do to show that no values of a, b exist? • Have you considered every possible type of example?

SUGGESTED LESSON OUTLINE

Introduction (10 minutes)

Explain the structure of the lesson to students.

Recall your work on irrational and rational numbers [last lesson].

You'll have a chance to review your work later today.

Today's lesson will help you improve your solutions.

Distribute mini whiteboards, pens, and erasers.

Write this statement on the class whiteboard:

The hypotenuse of a right triangle is irrational.

You'll be given a set of statements like this one. They are all about rational and irrational numbers. Your task is to decide whether the statement is always, sometimes or never true.

Model the lesson activity with students.

Ask students to spend a few minutes working alone or in pairs, to find an example of a right triangle, and show a calculation on their mini whiteboards of the length of the diagonal.

[Elsie], for your triangle, was the hypotenuse an irrational number?
So was this statement is true or false for your triangle?

If necessary, prompt students to think beyond their first examples.

What other side lengths could you try?

How about working backwards? Choose a rational number for the hypotenuse and see what happens.

What if the triangle has hypotenuse 5 units? Or $\sqrt{20}$ *units? What could the other side lengths be?*

Do you think this statement always, sometimes or never true? [Sometimes true.]

Now explore the reasoning involved in the task.

What did you need to do to show the statement was sometimes true? [Find an example for which the statement is true, and an example for which it is false. This is established with certainty once there is one example true, and one false.]

Some of the statements you will work with in this lesson are always or never true.

What would you need to show to be sure that a statement is always true or never true?

You're not expected to prove all the statements in this lesson. You do have to form conjectures, that is, decide what you think is correct with examples to support your conjecture.

To show a statement is always/never true requires proof. Proofs of some of the statements in this lesson are beyond the math available to many high school students.

Now summarize the task:

Try out examples of different numbers until you form a conjecture about whether the statement is always, sometimes or never true.

It's important to try lots of different examples.

Write your number examples, your conjecture, and your reasons for your conjecture on the task sheet.

Collaborative small group work: always, sometimes or never true? (20 minutes)

Organize students into groups of two or three. Explain the task.

Show students the projectable resource, *Poster with Headings*.

You are going to make a poster like this one. You'll see there are three columns, 'Always True', 'Sometimes True', and 'Never True'. You will be given some statements to classify on your poster.

Display the projector resource P-2, *Instructions for Always, Sometimes or Never True*.

These instructions explain how you are to work together to make your poster.

Clarify the instructions with students, then give each group a copy of the lesson task, *Always, Sometimes or Never True,* the sheet *Poster Headings,* a large sheet of paper for making a poster, scissors, and a glue stick.

Students collaborate to find examples illustrating the statements and form conjectures about them. They classify the statement as always, sometimes or never true on their poster, and write their reasoning on and around the statement.

During small group work, you have two tasks: to listen to students' collaborative work and to support student problem solving.

Listen to students' collaborative work.

Think about the range of examples students produce. Do they explore useful variation in numbers? Do they use integers, fractions, decimals, negative numbers, radicals, π, or very large numbers? Do students use expressions such $3-\sqrt{2}$? Students may have a limited notion of irrational numbers and rely on π and $\sqrt{2}$. Notice if students rely on truncated decimal representations, including calculator displays, and whether they experience any difficulties in manipulating radicals and fractions. Think about students' justifications. Do students have a clear grasp of the notion of a conjecture? Are they clear that finding some examples does not establish a statement is always true, but that finding examples for which the statement is both true and false establishes that the statement is sometimes true?

Support student problem solving.

Try not to make suggestions that resolve students' difficulties for them. Instead, ask questions to help students recognize errors, and question them to clarify their thinking.

In particular, prompt students to extend their range of examples. If, after prompting, students still use a limited range of numerical examples, distribute the hint sheet *Rational and Irrational Numbers*.

If students find it hard to get started on the task, suggest they choose one of these statements: 'The product of two rational numbers is irrational' or 'The product of a rational number and an irrational number is irrational'. Then prompt them to choose some numbers to try.

If several students in the class are struggling with the same issue, you could write a relevant question on the board. You might also ask a student who has performed well on a particular part of the task to help a struggling student.

If one student categorizes a particular card, challenge another student in the group to provide an explanation.

Cheryl categorized this card. Chan, why does Cheryl think this statement cannot be Always True/Sometimes True/Never True?

If the student is unable to answer this question, have the group discuss the work further. Explain that you will return in a few minutes to ask a similar question.

If students finish the task, give them the sheet *Extension Task*.

Towards the end of the session, identify one or two statements that several students have experienced difficulties in classifying. You can use these as the focus for the plenary discussion.

Whole-class discussion (15 minutes)

The intention in this discussion is that you work with examples that have been contentious in the class and help students clarify their reasoning about the task, rather than checking the truth of every statement in this session.

If necessary, distribute mini-whiteboards, pens and erasers again. For each of the statements there is a classification table as a projector resource, with space for examples for which the statement is true and for which it is false.

Choose one of the statements as the first focus of discussion and display the resource for that statement. Ask students to write on their whiteboards at least one example of a number illustrating the statement. Collect some examples to add to the table of results, asking students where they should be placed on the projection, and ask each contributing student to explain his/her reasoning.

[Rudy], is the statement is true or false for your number(s)? Explain that to me.

[Valentin] do you agree?

After several numerical examples, ask students to indicate on their mini-whiteboards whether they think the statement is always, sometimes, or never true.

[Eva], what is your conjecture?

We have three examples, for all of which the statement is true. Does that show that the statement is always true, for all numbers? [No, no particular examples can do that. We might always find a different number for which the statement is false.]

OK, so we suspect it's true, but we haven't proved it.

We've some numbers for which the statement is true, others for which it's false. Can we prove the statement is always true? Always false? [This statement is sometimes true.]

Many of the statements in this task are sometimes true although, for some statements, students may struggle to find examples of numbers that show this. Try to develop the difference between having examples that support a conjecture, and proving it.

We're conjecturing that this statement is always true – but maybe we just haven't found the right examples yet.

Prompt for new examples if students are reaching a incorrect conclusion.

So far all the examples we've found make the statement true/false. Any other numbers we can try?

I'm pretty sure there is a number that makes this statement true/false.

Ask students to vote to choose a second statement to discuss. Repeat the interactive work to establish whether the statement is sometimes, always or never true. If necessary, unresolved cases can be recorded as a task for students to pursue at a later date.

Next lesson: Review (15 minutes)

Invite students to review their work on rational and irrational numbers. Distribute students' original work on the assessment task, *Rational or Irrational?* Ask students to read their work and your questions about it.

If you have not added questions to individual pieces of work, then write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

You can now continue in one of two ways:

Either have the students revise their work on the original task, *Rational or Irrational?* Give each student a fresh copy of the original task and ask them use what they have learned in the lesson and through your questions to improve that original solution.

Or have students work on the related review task, *Rational or Irrational? (Revisited)*. Give each student a copy of that task and ask them answer the questions using their original solutions, your questions, and what they have learned in the lesson.

When a student is satisfied with his or her solution, ask them to make up a new statement about rational and irrational numbers, and answer it.

Some teachers ask students to do the review work for homework.

SOLUTIONS

Assessment task: *Rational or Irrational*?

1. Check that the numbers written are rational. Students tend to define rational numbers as **numbers that can be written as a fraction**.

   The problem with this is that irrational numbers such as $\frac{\sqrt{3}}{2}$ are also fractions.

   A better definition is that rational numbers are **numbers that can be written as a fraction of integers.**

2. Following from Q1, irrational numbers might be defined as **numbers that cannot be written as a fraction of integers.**

3. It is possible to find values of *a, b* to make **all** these statements true.

   a. It is possible to choose *a, b* such that the perimeter and area are both rational numbers.

      For example, choose a = 4 and b = 3 so the perimeter is 2(4 + 3)=14 units and the area is 4 × 3 = 12 square units.

   b. It is possible to choose *a, b* such that the perimeter is a rational number and the area irrational. The perimeter can be a sum of two irrational numbers in which the irrational parts have a zero sum.

   For example, if $a = 4 + \sqrt{6}$ and $b = 3 - \sqrt{6}$.

   The perimeter is $2(a + b) = 2(4 + \sqrt{6} + 3 - \sqrt{6}) = 2 \times 7 = 14$ which is rational.

   The area is $ab = (4 + \sqrt{6})(3 - \sqrt{6}) = 12 - 4\sqrt{6} + 3\sqrt{6} - 6 = 6 - \sqrt{6}$ which is irrational.

c. It is possible to choose *a, b* such that both perimeter and area are irrational numbers, if you choose, for example, different irrational roots.

For example, take $a = \sqrt{5}$ and $b = \sqrt{7}$.

Then $2(a+b) = 2(\sqrt{5} + \sqrt{7})$ which is irrational, and $ab = \sqrt{5} \text{ x } \sqrt{7} = \sqrt{35}$, which is also irrational.

d. It is possible to choose *a, b* such that the perimeter is an irrational number, and the area is a rational number. This occurs if *a, b* are multiples of the same irrational square root.

For example, $3\sqrt{5}$ and $\sqrt{5}$. You could also choose $a = b = \sqrt{5}$, say.

Collaborative small group work task

'The sum of a rational number and an irrational number is irrational'

This statement is always true.

An irrational number can be represented as a non-terminating, non-repeating decimal. Any rational number can be written in non-terminating repeating form. Adding the repeating pattern to the non-repeating decimal - this won't make it repeating. Alternatively, students might see this as a shift of the irrational number along the number line.

'The circumference of a circle is irrational'

This statement is sometimes true.

If the radius is rational, the circumference is irrational, because it is a rational multiple of π. If the radius is itself a fraction of π, the circumference can be rational.

For example, if $r = \frac{4}{\pi}$, then $C = 2\pi r = 2\pi \times \frac{4}{\pi} = 8$

'The diagonal of a square is irrational'

This statement is sometimes true.

If the sides of the square are 2 units long, then the diagonal is $\sqrt{2^2 + 2^2} = \sqrt{4+4} = \sqrt{8}$ which is irrational.

If the sides of the square are $\sqrt{8}$ units, then the diagonal of the square is $\sqrt{(\sqrt{8})^2 + (\sqrt{8})^2} = \sqrt{8+8} = \sqrt{16} = 4$.

'The sum of two rational numbers is rational'

This statement is always true.

The rational numbers are closed under addition: you can't make an irrational number by adding two rationals.

'The product of a rational number and an irrational number is irrational'

This statement is sometimes true.

Multiply any irrational number by the rational number zero. The product is rational.

Choose any other rational with any irrational number, and the product is irrational.

‘The sum of two irrational numbers is irrational’

This statement is sometimes true.

If the irrational parts of the numbers have zero sum, the sum is rational. If not, the sum is irrational. It is true, for example, for $2\sqrt{7},\ \sqrt{3}$. It is false, for example, for $3+2\sqrt{5},-2\sqrt{5}$.

‘The product of two rational numbers is rational’

This is true for all rational numbers: the rationals are closed under multiplication.

If *a, b, c, d* are integers with *b, d* non-zero, then $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$ is also rational. If *b* or *d* is zero, the product is not defined, rather than irrational.

‘The product of two irrational numbers is irrational’

This statement is sometimes true.

For example, $\sqrt{2}\times\sqrt{8}$ is rational, but $\sqrt{3}\times\sqrt{8}$ is irrational.

Extension Task

‘An expression containing both $\sqrt{6}$ and π is irrational’

This statement is sometimes true.

For example, $\sqrt{6}+\pi$ is irrational but $\frac{\sqrt{6}\pi}{\sqrt{24}\pi}=\frac{\sqrt{6}}{2\sqrt{6}}=\frac{1}{2}$ is rational.

Students might feel that including expressions not in their simplest form is ‘cheating’. It may help to emphasize that these are just different ways of writing the same number.

‘Between two rational numbers there is an irrational number’

This statement is always true.

Students might try to show this by providing a method for finding an irrational. For example, one could first find the difference *d* between the two rationals. This will be rational. Then one could add an irrational fraction that is smaller than this difference (e.g. $\sqrt{0.5d}$) to the smaller rational. The result will be an irrational that lies between the two.

Alternatively, represent the two rationals by two repeating non-terminating decimals, for example:

0.12121212... and 0.123123123...

Find the first decimal place at which they differ. In this case it is the third digit after the decimal point. The digits are 1 and 3 respectively.

Use all the digits so far to start the new number, in this case 0.12. Add one to the third digit of the smaller number, in this case the third digit of the first number is changed from 1 to 2. Replace the digits after this 2 with a non-repeating, non-terminating decimal (e.g. all the digits of π):

0.12214159265358 ....

This method does not always work. Sometimes when 1 is added to the first differing digit, the resulting number is bigger than both numbers. For example

0.12121212... And 0.122122122...

In this case, add 1 to the next digit of the smaller number, 1 is added to the fourth digit. Then again the digits after this are replaced with a non-repeating, non-terminating decimal:

0.121314159265358 ...

‘Between two irrational numbers there is an irrational number’

This statement is always true.

Students might try to show this by example. They may take the average of two irrational numbers. This will always be irrational, unless the irrational numbers sum to zero, e.g. π and $-\pi$. In this case, they could divide the larger irrational number by a rational, say 2. This will result in an irrational number between the two rational ones.

‘If the circumference of a circle is rational, then the area is rational’

This statement is never true.

This is because the circumference and area involve different multiples of π.

π is a transcendental (non-algebraic) number, so it cannot be the solution of a polynomial equation. You cannot use multiples of different powers of π to give a rational solution.

We are told that C is rational, so write $C = k$ for some rational number k.

We know $C = k = 2\pi r$.

So $r = \frac{k}{2\pi}$ which must be irrational.

Then $A = \pi r^2 = \pi \times (\frac{k}{2\pi})^2 = \frac{\pi \times k^2}{4\pi^2} = \frac{k^2}{4\pi}$ which is irrational.

‘If the area of a circle is rational, then the circumference is rational’

This statement is never true.

This is because multiples of different powers of π cannot give a rational solution.

A is rational, so write $A = j$ for some rational number j.

$A = j = \pi r^2$. So $r^2 = \frac{j}{\pi}$ and $r = \frac{\sqrt{j}}{\sqrt{\pi}}$.

Then $C = 2\pi r = 2\pi \frac{\sqrt{j}}{\sqrt{\pi}} = 2\sqrt{j} \times \sqrt{\pi}$.

So C is a rational or an irrational multiple of $\sqrt{\pi}$.

Students could gain the sense of the last two examples without an algebraic proof or discussion of transcendental numbers if you ask them to pick a value for r and then try to adjust to make the statements true.

Assessment task: *Rational or Irrational*? (*revisited*)

1. These numbers are rational:

$$3 - 27 \qquad \frac{\sqrt{3}}{\sqrt{27}} \qquad (\sqrt{27} + 3)(\sqrt{27}$$

Students who claim that $\frac{\sqrt{3}}{27}$ is rational may have incomplete definitions of rational and irrational numbers, referring to fractions rather than fractions of integers, or ratios of numbers rather than ratios of integers. Students who do not pick out the products and quotients involving expressions with radicals may need help in manipulating radicals.

2a. Fubara is incorrect. The student might show this by choosing two irrational numbers e.g. $\sqrt{5}$ $3 - \sqrt{5}$ whose irrational parts sum to zero, and thus which have a rational sum.

2b. Nancy is incorrect. The student might show this by choosing two irrational numbers whose product is rational. For example, $(3 + \sqrt{5})(3 - \sqrt{5}) = 9 - 5 = 4$.

3. Complete the table

'If you divide one irrational number by another, the result is always irrational.'

This is false. For example, $\frac{\sqrt{3}}{\sqrt{3}} = 1$.

'If you divide a rational number by an irrational number, the result is always irrational.'

This is false. If the rational number is zero, the quotient is always rational. Otherwise the result is always irrational.

'If the radius of a circle is irrational, the area must be irrational.'

This is false. For example, let $r = \frac{7}{\sqrt{\pi}}$. Then $A = \pi r^2 = \pi \times \left(\frac{7}{\sqrt{\pi}}\right)^2 = \frac{49\pi}{\pi} = 49$.

Rational or Irrational?

1. a. Write three rational numbers.

   ....

   b. Explain what a rational number is in your own words.

   ....

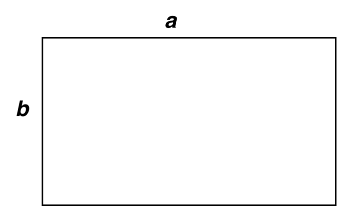
2. a. Write three irrational numbers.

   ....

   b. Explain what an irrational number is in your own words.

   ....

3. This rectangle has sides lengths a and b.



   Decide if it is possible to find a and b to make the statements below true.

   If you think it is possible, give values for a and b.

   If you think it is impossible, explain why no values of a and b will work.

   a. The perimeter and area are both rational numbers.

   ....

   ....

   b. The perimeter is a rational number, and the area is an irrational number.

   ....

   ....

   c. The perimeter and area are both irrational numbers.

   ....

   ....

   d. The perimeter is an irrational number, and the area is a rational number.

   ....

   ....

Always, Sometimes or Never True

The sum of a rational number and an irrational number is irrational.	The circumference of a circle is irrational.
The diagonal of a square is irrational.	The sum of two rational numbers is rational.
The product of a rational number and an irrational number is irrational.	The sum of two irrational numbers is irrational.
The product of two rational numbers is irrational.	The product of two irrational numbers is irrational.

Rational and Irrational Numbers

6	$\frac{5}{6}$	$\sqrt{6}+\sqrt{3}$
$1-\pi$	$5+\sqrt{6}$	$4+\sqrt{3}$
π	$\frac{\pi}{2}$	$\frac{\sqrt{6}}{\sqrt{3}}$
$\sqrt{6}$	$5-\sqrt{6}$	$0.\overline{6}$
0.45	-6	$\frac{\sqrt{3}}{\pi}$

Poster Headings

Always True	Sometimes True	Never True

Extension Task

An expression containing both $\sqrt{6}$ and π is irrational.	Between two rational numbers there is an irrational number.
Between two irrational numbers there is an irrational number.	If the area of a circle is rational, then the circumference is rational.
If the circumference of a circle is rational, then the area is rational.	Make up your own statement.

Rational or Irrational? (revisited)

1. Circle any of these numbers that are rational.

$\frac{\sqrt{3}}{27}$ $\qquad 3-27 \qquad \sqrt{3}+\sqrt{27} \qquad \frac{\sqrt{3}}{\sqrt{27}} \qquad (\sqrt{27}+3)(\sqrt{27}-3)$

2a. Fubara says, “The sum of two irrational numbers is always irrational.”

Show that Fubara is incorrect.

....

....

b. Nancy says, “The product of two irrational numbers is always irrational.”

Show that Nancy is incorrect.

....

....

3. Complete the table. Make sure to explain your answer.

Statement	**True or false**	**Explanation**
If you divide one irrational number by another, the result is always irrational.		
If you divide a rational number by an irrational number, the result is always irrational.		
If the radius of a circle is irrational, the area must be irrational.		

Poster with Headings

Rational and Irrational Numbers

Always true	Sometimes true	Never true

The product of two rational numbers is irrational.

Writing your reasoning on the poster.

Your names

Instructions for *Always, Sometimes* or *Never True*

1. Choose a statement.
   - Try out different numbers.
   - Write your examples on the statement card.

2. Conjecture: decide whether you think each statement is always, sometimes or never true.
   - **Always true**: explain why on the poster.
   - **Sometimes true**: write an example for which it is true and an example for which it is false.
   - **Never true**: explain why on the poster.

Always, Sometimes or Never True?

The sum of a rational number and an irrational number is irrational.	
True for:	False for:

Always, Sometimes or Never True?

The circumference of a circle is irrational.

True for:	False for:

Always, Sometimes or Never True?

The diagonal of a square is irrational.

True for:	False for:

Always, Sometimes or Never True?

The sum of two rational numbers is rational.

True for:	False for:

Always, Sometimes or Never True?

The product of a rational number and an irrational number is irrational.

True for:	False for:

Always, Sometimes or Never True?

The sum of two irrational numbers is irrational.	
True for:	False for:

Always, Sometimes or Never True?

The product of two rational numbers is irrational.

True for:	False for:

Always, Sometimes or Never True?

The product of two irrational numbers is irrational.

True for:	False for:

Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
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based on their observation of trials in US classrooms
along with comments from teachers and other users.

This project was conceived and directed for
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by
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and based at the University of California, Berkeley

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